PARTIAL DERIVATIVES' NOTATION

FOR $f(x,y) = x^4y^2 + xe^y$ AND $z = x^4y^2 + xe^y$

$$f_x(x,y) = 4x^3y^2 + e^y \qquad \frac{\partial z}{\partial x} = 4x^3y^2 + e^y$$

$$f_y(x,y) = 2x^4y + xe^y \qquad \frac{\partial z}{\partial y} = 2x^4y + xe^y$$

$$(f_x)_x = f_{xx}(x,y) = 12x^2y^2 \qquad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x^2} = 12x^2y^2$$

$$(f_y)_y = f_{yy}(x,y) = 2x^4 + xe^y \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y^2} = 2x^4 + xe^y$$

$$(f_x)_y = f_{xy}(x,y) = 8x^3y + e^y \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial y\,\partial x} = 8x^3y + e^y$$

So $f_{xy} = \dfrac{\partial^2 z}{\partial y\,\partial x}$

NOTE HOW THE ORDER IS REVERSED!

For $u = f(x,y,z)$,

$$f_{xzy}(x,y,z) = \frac{\partial^3 u}{\partial y\,\partial z\,\partial x}$$

(See p. 918)

second partial derivatives

If $z = f(x, y)$, we use the following

$$(f_x)_x = f_{xx} = f_{11} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 z}{\partial x^2}$$

$$(f_x)_y = f_{xy} = f_{12} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial^2 z}{\partial y\,\partial x}$$

$$(f_y)_x = f_{yx} = f_{21} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial^2 z}{\partial x\,\partial y}$$

$$(f_y)_y = f_{yy} = f_{22} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 z}{\partial y^2}$$

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